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On the completeness of classical electromagnetism

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Abstract

The possibility of an incompleteness of the equations of electromagnetism is analyzed using a thought experiment that shows a non-physical behavior according to classical electromagnetism. Basically, from Maxwell equations it is shown that a particular passive, isolated circuit could present a growth of its currents. Resolution of this problem is sought within the context of the usual electromagnetism and also using the possibly simplest generalization of Maxwell equations, a reduced version of Ohmura equations.

I. Introduction

Maxwell equations provide a very satisfactory model of classical electromagnetic phenomena. Although some inconsistencies have been pointed out, they are attributed to incomplete mathematical solutions and problems of interpretation.[1] The question however arises as to what point can Maxwell equations be considered complete, capable of describing every electromagnetic phenomenon. From a purely theoretical point of view extensions of Maxwell equations have been proposed, for instance to include magnetic sources analogous to the electric ones (magnetic charges and currents), and to account for a finite photon mass (Proca equations). Besides, when expressed in the formalism of classical spinor fields, Maxwell equations have a zero complex scalar component. The extension to include a non-zero scalar component corresponds to Ohmura equations (which additionally include magnetic sources).[2] Experimental evidence has set very small limits to the magnetic charges of known particles and to the photon mass.[3] The situation with respect to the scalar components, however, is worth further study. The purpose of the present work is to show that at least in a thought experiment a non-physical behavior could be obtained from a naive use of Maxwell equations. Possible resolutions of this problem are then considered both, within the usual Maxwell equations, and with a reduced version of Ohmura equations. The basic idea is simple to appreciate considering the circuits shown in Fig. 1, consisting each in a series solenoid-capacitor. The capacitor of each of the circuits lies inside the solenoid of the other. Due to the solenoids being closed the mutual inductance between circuits is ideally zero so that, at the level of a quasi-stationary approximation, one expects independent LC oscillations in each circuit. It is seen that the time dependent toroidal magnetic field generated by the charging-discharging capacitor induces an electromotive force in the solenoid. This can be expressed as an effective mutual inductance between circuits. At variance with a truly inductive coupling, the sign of the effective mutual inductance coefficient can be made different for each circuit by proper connection. The resulting coupled equations for the capacitor charges have then solutions showing growth with time for almost every initial condition.

II. Analysis

In Fig. 1a both circuits are shown, with the capacitors outside the solenoids, and without their internal dielectric for clarity. Note that in circuit 2 the upper plate of the capacitor is connected to the cable leaving the solenoid from below, which is the only difference between both circuits (both solenoids are equally built, so as the capacitors). In Fig. 1b an inside view of a solenoid with the internal capacitor is shown, denoting also the linear material of relative permittivity and permeability ε_{rel} and μ_{rel}, respectively. Although the wires in the solenoids are drawn well separated for clarity, the N turns in each solenoid are considered closely packed so that ideally non magnetic field is generated outside the solenoids.

One expects in this system an oscillation of frequency ν, which can in principle be made sufficiently small for large enough values of ε_{rel} and μ_{rel} that control the capacitance and inductance of the system. In the following we analyze the quasi-stationary behavior of the circuit valid at small ν. More precisely the small parameter is $\lambda \equiv \nu l_s/c$, where c is the speed of light in the material medium and l_s a characteristic length of the system. We will require further that the currents $i_{1,2}$ are uniform along the whole cable (connecting plus solenoid) of the corresponding circuit, which requires $\lambda_0 \equiv \nu l_c/c_0 \ll 1$, where l_c is the length of the cable and c_0 the speed of light in vacuum.

From Ampère law

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \tag{1}$$

the magnetic field $\mathbf{H}$ is generated by the free current density $\mathbf{j}$ and the time variation of the displacement vector field $\mathbf{D}$. The contributions to $\mathbf{j}$ in each circuit are composed of the main current in the solenoid and connecting cables, which gives rise to the currents $i_{1,2}$, and the contributions, denoted as secondary, due to currents in the charging-discharging capacitor plates together with the induced currents (Foucault currents) in the solenoid and plates, to ensure zero electric and magnetic fields inside these conductors, assumed of negligible resistivity (in the sense that the penetration length is much smaller than the conductors thickness).

From Gauss and charge conservation laws

$$\begin{aligned} \boldsymbol{\nabla} \cdot \mathbf{D} &= \rho, & (2)\\ \boldsymbol{\nabla} \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} &= 0, & (3) \end{aligned}$$

with ρ the free charge density distributed in the capacitor plates and induced in the solenoid internal surface, using also the linearity of the problem, one can see that the contributions to $\mathbf{H}$ at order λ coming from the time variation of $\mathbf{D}$ and the secondary currents scale as, for instance, the instantaneous total charge in the plate of the capacitor, indicated in Fig. 1. In effect, by the linearity of the system one can split the problem of calculating the magnetic field inside the solenoid into that of the solenoid with its main current containing an uncharged capacitor, and that of a charging-discharging capacitor inside a conducting toroid without its main current. In the first problem, as proved simply using the azimuthal symmetry valid in this case, the magnetic field outside the capacitor plates is just the one inside an empty solenoid, as the Foucault currents induced in the capacitor plates only cancel the magnetic field inside these conductors, generating no field outside them. Besides, at order λ (quasi-stationary approximation) no free surface charge density is induced in a conductor by the alternating magnetic field,[4] so that no charge appears in the capacitor plates and no conservative part of $\mathbf{D}$ is generated. The effect of the $\mathbf{D}$ field can then be neglected for this problem as, from Eq. (2) and Faraday law (for the electric field $\mathbf{E}$ and magnetic induction $\mathbf{B}$)

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{4}$$

$\mathbf{E}$ (and consequently $\mathbf{D}$) scales as the characteristic frequency of oscillation ν, so that its contribution to $\mathbf{H}$ scales as ν^2, and can be neglected at order λ (more precisely, one has $|\partial \mathbf{D}/\partial t| \sim \lambda^2 \, |\mathbf{j}|$).

In a similar manner, in the second problem (charging-discharging capacitor and no main current in the solenoid) the magnetic field scales, according to Eqs. (1) and (3), as the characteristic frequency of oscillation ν, and so, from Eq. (4), the curl of $\mathbf{E}$ (and consequently of $\mathbf{D}$) scales as ν^2 (one has $|\boldsymbol{\nabla} \times \mathbf{D}| \sim \lambda^2 \, |\boldsymbol{\nabla} \cdot \mathbf{D}|$). At the order λ considered this problem is then well approximated by a quasi-electrostatic one in which $\mathbf{D}$ scales as the instantaneous charge distribution, quantified, for instance, by the charge in one plate of the capacitor. As the currents in the capacitor plates and those induced in the solenoid also scale in the same way, Ampère equation (1) indicates that the corresponding contribution to $\mathbf{H}$ scales as $dQ_{2,1}/dt$ for each circuit.

One can thus write for the azimuthal component of the magnetic field at any (r, φ, z) inside the solenoid of either circuit,

$$H_{\varphi 1,2} = \pm \frac{N \, i_{1,2}}{2\pi r} + \frac{dQ_{2,1}}{dt} F\left(r, \varphi, z\right), \tag{5}$$

where the first term corresponds to the contribution form the main current, N is the number of turns of each solenoid, $\pm$ corresponds to the sign of j_z at the inner radius of the solenoid, and $F\left(r,\varphi,z\right)$ is a function that depends only on the geometry and material characteristics, but not on the charge of the capacitor (an approximate expression of F will be considered later).

One can now evaluate Faraday law in circuital form

$$\oint_{C_{1,2}} \mathbf{E}\cdot d\mathbf{l}+\frac{d\phi_{1,2}}{dt}=0, \tag{6}$$

taking as the curve $C_{1,2}$ in each circuit the total extension of the corresponding cable (connecting plus solenoid) which is made closed along a path going form one plate of the capacitor to the other (sufficiently close to the cable connecting the plates so as to concatenate no magnetic flux. Alternatively, the path could be closed across the cables connecting the capacitor plates just before entering the solenoid. The result is the same because the electric field inside the conductors is zero). The direction of circulation assumed is that of the currents shown in Fig 1, and the connecting cables are close together. In these conditions there is magnetic flux across $C_{1,2}$ only inside the solenoids, which can be evaluated as

$$\begin{aligned}
\phi_1 &= \mu_0\frac{N}{2\pi}\int_0^{2\pi} d\varphi\left[\int_{So} H_{\varphi 1}dS+\mu_{rel}\int_{Sd} H_{\varphi 1}dS\right],\\
\phi_2 &= -\mu_0\frac{N}{2\pi}\int_0^{2\pi} d\varphi\left[\int_{So} H_{\varphi 2}dS+\mu_{rel}\int_{Sd} H_{\varphi 2}dS\right],
\end{aligned}$$

where μ_0 is the permeability of the vacuum, S_o denotes the area outside the capacitor in each section $\varphi = const$ inside the solenoid, and Sd represents the corresponding area of the dielectric (the cross section of the capacitor plates are not considered because of the assumption of ideal conductors without varying magnetic field inside them). The difference in sign is due to the convention of circulation assumed.

From Eq. (5), defining the inductance L and effective mutual inductance

M as

$$L \equiv \mu_0 N^2 \left[\int_{So} \frac{1}{r} dS + \mu_{rel} \int_{Sd} \frac{1}{r} dS \right], \tag{7}$$

$$\begin{aligned} M &\equiv \mu_0 \frac{N}{2\pi} \int_0^{2\pi} d\varphi \left[\int_{So} F(r, \varphi, z)\, dS + \right. \\ &\quad \left. \mu_{rel} \int_{Sd} F(r, \varphi, z)\, dS \right], \end{aligned} \tag{8}$$

one finally has (using that $dQ_{1,2}/dt = i_{1,2}$)

$$\begin{aligned} \phi_1 &= L\, i_1 + M\, i_2, \\ \phi_2 &= L\, i_2 - M\, i_1. \end{aligned}$$

As the cables are assumed of zero resistivity, only the path across the capacitor plates contribute to the electric field circulation in Eq. (6) so one can write

$$\oint_{C_{1,2}} \mathbf{E} \cdot d\mathbf{l} = \frac{Q_{1,2}}{C},$$

where C is the capacitance of each capacitor (evaluated of course considering that they are inside the closed conducting surface of the solenoids). As mentioned above, no charge is induced in the plates by the alternating magnetic field, so that only the charges $Q_{1,2}$ contribute to the electric field considered.

Eq. (6) gives then rise to the pair of coupled equations

$$L \frac{di_1}{dt} + M \frac{di_2}{dt} + \frac{Q_1}{C} = 0, \tag{9a}$$

$$L \frac{di_2}{dt} - M \frac{di_1}{dt} + \frac{Q_2}{C} = 0. \tag{9b}$$

This system has a general solution of the form (i is the imaginary unit)

$$Q_{1,2}(t) = A_{1,2} \exp[i\xi t] + B_{1,2} \exp[-i\xi t], \tag{10}$$

where $A_{1,2}$ and $B_{1,2}$ are constants, and

$$\xi \equiv \sqrt{\frac{L + iM}{C\left(L^2 + M^2\right)}}.$$

As one expects $M \sim \lambda L \ll L$ one has

$$\xi \simeq \omega\,(1 + i\gamma)\,,$$

where

$$\begin{aligned} \omega &\equiv \frac{1}{\sqrt{LC}}, \\ \gamma &\equiv \frac{M}{2L} \ll 1. \end{aligned}$$

One thus obtains the usual LC oscillation plus an exponential growth of capacitor charges and currents at a rate $\gamma\omega$. This effect is due to the seeming lack of a mechanism to counteract the induction of a current in the solenoid of one circuit by the charging-discharging capacitor in the other circuit.

III. Parameter estimation

We now estimate the different parameters in the circuit. This can be done easily in the case of azimuthal symmetry, which requires that the currents in the capacitor plates have this symmetry. This can be achieved quite closely by making each capacitor plate in a given circuit as a tight spiral of the same cable connecting to the solenoid in that circuit. The non-symmetrical contribution to the magnetic field from the current in the short vertical extension of wire connecting the plates can be evaluated explicitly using Biot-Savart expression, resulting in about $10 - 20\%$ of that generated by the currents in the capacitor plates (for $d \lessapprox b - a$), so that it can be neglected in the estimations (besides, this small contribution adds up constructively to the one considered). There are also poloidal currents induced in the internal surface of the solenoid by the moving charges in the capacitor plates. It is easy to see that these currents either do not contribute to the effect if they circulate on the largest radius surface, or contribute increasing the effect if they circulate on the shortest radius surface. So they are not included in the estimations.

What is then needed is the contribution to $H_{\varphi 1,2}$ generated by the charging-discharging capacitor inside the conducting surface of the solenoid without its main current.

As mentioned above, the evaluation of $\mathbf{D}$ for this case can be made as for an electrostatic problem corresponding to the instantaneous charge distribution in the conductors. This is a simple standard problem if end effects are neglected, assuming also homogeneous surface charge densities in the inner and outer surfaces of the capacitor plates and in the inner surface of the solenoid. These approximations were verified solving numerically the non-approximated problem with the commercial software QuickField,[5] which shows that the approximation is very good for d and d_0 smaller than $b-a$, and gets better the larger the value of ε_{rel}.

In the evaluations to follow the distance to the solenoid from all sides of the capacitor was taken as d_0 in order not to introduce new symbols.

The result for $\mathbf{D}$ is, inside the capacitor,

$$\mathbf{D}_{1,2} = -\frac{\varepsilon_{rel}\varepsilon_0 Q_{1,2}\left(t\right)}{d\,C}\mathbf{e}_z,$$

while outside it is

$$\mathbf{D}_{1,2} = \frac{\varepsilon_0 Q_{1,2}\left(t\right)}{2d_0\,C}\mathbf{e}_z.$$

In these expressions ε_0 is the vacuum permittivity, $\mathbf{e}_z$ the unit vector in the z direction, and C is the capacitor capacitance

$$C = \varepsilon_0\pi\left(b^2-a^2\right)\left(\frac{\varepsilon_{rel}}{d}+\frac{1}{2d_0}\right). \tag{11}$$

From Eq. (1), using the azimuthal symmetry, one obtains for the corresponding contribution to $H_{\varphi 1,2}$, denoted as $H'_{\varphi 1,2}$,

$$H'_{\varphi 1,2} = -\frac{\varepsilon_{rel}\left(r^2-a^2\right)}{2\pi r d\left(b^2-a^2\right)\left(\frac{\varepsilon_{rel}}{d}+\frac{1}{2d_0}\right)}\frac{dQ_{1,2}}{dt},$$

inside the capacitor, and

$$H'_{\varphi 1,2} = \frac{\left(r^2-a^2\right)}{4\pi r d_0\left(b^2-a^2\right)\left(\frac{\varepsilon_{rel}}{d}+\frac{1}{2d_0}\right)}\frac{dQ_{1,2}}{dt},$$

outside the capacitor. The factor that multiplies $dQ_{1,2}/dt$ is the function $F\left(r,\varphi,z\right)$ defined in expression (5). From the definitions (7) and (8) one thus have

$$\begin{aligned} L \;&=\; \frac{\mu_0 N^2 d}{2\pi}\left\{\mu_{rel}\ln\left(\frac{b}{a}\right)+\right. \\ &\left.\ln\left[\frac{a\left(b+d_0\right)}{b\left(a-d_0\right)}\right]+\frac{2d_0}{d}\ln\left(\frac{b+d_0}{a-d_0}\right)\right\}, \end{aligned}$$

and

$$M = \frac{\mu_0 N \left(\mu_{rel}\varepsilon_{rel} - 1\right)}{4\pi\left(\frac{\varepsilon_{rel}}{d} + \frac{1}{2d_0}\right)}\left[1 - \frac{2\ln(b/a)}{(b/a)^2 - 1}\right].$$

Note that M is zero for $\mu_{rel}\varepsilon_{rel} = 1$, as the effect of $\partial\mathbf{D}/\partial t$ outside the capacitor cancels that inside it. This is another reason for the use of a medium inside the capacitor.

For high enough values of μ_{rel} and ε_{rel} one has the simpler expressions

$$\begin{aligned} L &= \frac{\mu_0\mu_{rel}N^2 d}{2\pi}\ln\left(\frac{b}{a}\right), \\ M &= \frac{\mu_0\mu_{rel}Nd}{4\pi}\left[1 - \frac{2\ln(b/a)}{(b/a)^2 - 1}\right], \end{aligned}$$

and the corresponding value of $\gamma = M/(2L)$

$$\gamma = \frac{1}{4N\ln(b/a)}\left[1 - \frac{2\ln(b/a)}{(b/a)^2 - 1}\right],$$

and of the frequency

$$\nu = \frac{c_0}{\pi N a\sqrt{2\mu_{rel}\varepsilon_{rel}\left[(b/a)^2 - 1\right]\ln(b/a)}}.$$

At the same level of approximation, taking $l_s = b$, one has for the small parameters $\lambda \equiv \nu l_s/c$ and $\lambda_0 \equiv \nu l_c/c_0$,

$$\lambda = \frac{1}{\pi N\sqrt{2\left[1 - (a/b)^2\right]\ln(b/a)}},$$

and

$$\lambda_0 = \frac{l_c}{\pi N\sqrt{2\mu_{rel}\varepsilon_{rel}\left(b^2 - a^2\right)\ln(b/a)}}.$$

As expected, γ and λ are of the same order, and the expression of λ shows that the condition $\lambda \ll 1$ is easily achievable in practice. On the other hand, since the cable length l_c scales as $Nl_s \sim Nb$ one has

$$\lambda_0 \sim \frac{1}{\sqrt{\mu_{rel}\varepsilon_{rel}\left[1 - (a/b)^2\right]\ln(b/a)}},$$

so that a small enough λ_0 can be achieved only by a high value of $\mu_{rel}\varepsilon_{rel}$.

IV. Inclusion of losses

So far the conductors and media were considered losseless. If the resistivity of the conductors is included the electric field circulation in Eq. (6) along the cables contributes a term $Ri_{1,2}$ for the corresponding circuit, with R the resistance of the whole cable length l_c. The circuit equations (9) are then replaced by

$$\begin{aligned} L\frac{di_1}{dt} + M\frac{di_2}{dt} + Ri_1 + \frac{Q_1}{C} &= 0, \\ L\frac{di_2}{dt} - M\frac{di_1}{dt} + Ri_2 + \frac{Q_2}{C} &= 0. \end{aligned}$$

This system has a general solution of the form

$$Q_{1,2}(t) = A_{1,2}\exp\left[\xi^+ t\right] + B_{1,2}\exp\left[\xi^- t\right],$$

with

$$\xi^\pm = \frac{-R/L \pm \sqrt{(R/L)^2 - 4(1+2i\gamma)\,\omega^2}}{2(1+2i\gamma)},$$

where, as before, $\omega = 1/\sqrt{LC}$. For $\gamma \ll 1$ this expression can be approximated by

$$\xi^\pm = \pm i\omega - \frac{R}{2L} \pm \gamma\omega,$$

so, apart from the LC oscillation at angular frequency ω, a growth in capacitor charges and currents is expected if

$$R < 2\gamma\omega L,$$

at a rate $\gamma\omega - R/(2L)$.

Losses occur also in the material medium. If it is characterized by an electrical conductivity σ, and complex permittivity and permeability

$$\begin{aligned} \varepsilon &= \varepsilon' + i\varepsilon'', \\ \mu &= \mu' + i\mu'', \end{aligned}$$

the average local power loss per unit volume is given by[4]

$$w^- = \frac{\sigma}{2}\left|\mathbf{E}_0\right|^2 + \frac{\omega}{2}\left(\varepsilon''\left|\mathbf{E}_0\right|^2 + \mu''\left|\mathbf{H}_0\right|^2\right),$$

where $\mathbf{E}_0$ and $\mathbf{H}_0$ are the local amplitude of the oscillating electric and magnetic fields. On the other hand, the local average electromagnetic energy per unit volume is

$$u = \frac{1}{4}\left(\varepsilon' \left|\mathbf{E}_0\right|^2 + \mu' \left|\mathbf{H}_0\right|^2\right),$$

so that the average local power "gain" per unit volume due to the anomalous growth is $w^+ = 2\gamma\omega u$ ($R \ll 2\gamma\omega L$ is assumed for simplicity), that is,

$$w^+ = \frac{\gamma\omega}{2}\left(\varepsilon' \left|\mathbf{E}_0\right|^2 + \mu' \left|\mathbf{H}_0\right|^2\right).$$

Net power gain ($w^- < w^+$) is then possible in principle if

$$\begin{aligned} \varepsilon'' + \sigma/\omega &< \gamma\varepsilon', \\ \mu'' &< \gamma\mu'. \end{aligned}$$

Additional losses occur due to the Foucault currents induced in the capacitor plates to ensure zero magnetic field inside them as the solenoid current oscillates (the resistivity is considered finite but small so that the penetration length of electric and magnetic fields is small compared to the conductors thickness). As these surface currents are of the same order as those in the solenoid (they have similar penetration length, extend over surfaces of similar extension, and generate magnetic fields of the same order), the total power loss in each capacitor plate is similar to that in the solenoid, of average $R\, i_0^2/2$, where i_0 is the amplitude of the oscillating current.

Finally, radiation losses are in general of order λ^2. In the conditions considered, $\lambda \ll 1$, the radiation is mainly dipolar[3] and can be estimated to be a very negligible fraction of the average power in the circuit.

In this way, the effect is easily prevented if the resistance in the cables is high or the material too lossy. However, if granular materials with sufficiently high values of ε' and μ', and low values of σ, ε'', and μ'' can be synthesized, which is possible in principle,[6] and good conductors employed, the thought experiment could be actually performed and the existence or not of the effect verified. Note that the evaluation of R must include the skin effect of the oscillating currents, dependent on ω, so that careful design is required for the effect to show up when real elements are used. What is meant by this is that the thought experiment can in principle be falsified by an actual experiment.

V. Possible paradox resolutions

We are thus faced with the paradoxical result that a particularly simple, isolated, passive device could increase its energy with time. This non-physical behavior cannot be explained, for instance, in terms of advanced electromagnetic waves (even if the existence of this kind of waves is accepted), as radiation effects are of higher order, in the small parameter λ, than the predicted effect.

A first possible resolution is considered within the frame of the usual electromagnetism. The point is that in the problem of the solenoid with its main current containing the uncharged capacitor it was assumed that no electric charge was induced on the conducting surfaces of the capacitor due to the time varying magnetic field, a known result valid in the quasi-stationary approximation.[4] On the other hand, the possible induction of a surface electric charge in a current carrying conductor in a static magnetic field (so allowed to pervade the interior of the conductor) is also a well known effect (Hall effect). In particular, the set in motion of a very good conductor originally at rest in a constant magnetic field generates an electric surface polarization which ensures the condition of zero electric field inside the conductor (in the conductor frame of reference, in which, by the way, the original value of the constant magnetic field persists, inside the conductor as well).

In this way, if one were to admit that the magnetic field of the solenoid penetrates the conducting plates of the capacitor, and that the zero electric field condition inside them is ensured by a proper surface charge distribution in the plates, and additional contribution to $\oint_{C_{1,2}} \mathbf{E} \cdot d\mathbf{l}$ in Eq. (6) would result from the electric field generated by these charges in the path between the capacitor plates. This contribution can be easily estimated using Eq. (6) for a closed curve that includes the path considered across the capacitor plates and which is made close along the interior of both plates and a second path between them in a region where the electric field generated by the charge distribution can be considered very small (at $r \approx (a+b)/2$). The result is that these new contributions can be written as

$$\begin{aligned}\oint_{C_1} \mathbf{E}' \cdot d\mathbf{l} &= M' \frac{di_2}{dt}, \\ \oint_{C_2} \mathbf{E}' \cdot d\mathbf{l} &= -M' \frac{di_1}{dt},\end{aligned}$$

where

$$M' \approx \frac{\mu_0 \mu_{rel} N d}{2\pi} \ln\left(\frac{2b}{a+b}\right).$$

The addition of these contributions is equivalent to replace M in Eqs. (9) by $M - M'$. As the estimations of M and M' give values of the same order one could hope to cancel the anomalous effect by obtaining $M = M'$ in a rigorous way.

This resolution is very attractive as the original equations of electromagnetism are considered. It is still problematic to explain properly the reason why the alternating magnetic field penetrates the conductors and so forces the apparition of the surface charges.

The second possible resolution could require an extension of Maxwell equations. Let us analyze with some detail that proposed by Ohmura fifty years ago[2] based on the spinor formalism of classical fields. In particular, we consider the simplest version of Ohmura equations, without magnetic monopole sources, and with solenoidal magnetic induction $\mathbf{B}$ (even in the absence of magnetic monopoles, $\boldsymbol{\nabla} \cdot \mathbf{B}$ can be non-zero by the presence of a pseudo-scalar function h in the most general Ohmura equations). These equations, formulated in vacuum, reduce to

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} + \frac{\partial e}{\partial t}, \tag{12a}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0, \tag{12b}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{12c}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) - \boldsymbol{\nabla} e, \tag{12d}$$

where e is a scalar field. Taking the time derivative of the extended Gauss law and the divergence of the extended Ampère law one obtains after some little manipulation

$$\mu_0 \varepsilon_0 \frac{\partial^2 e}{\partial t^2} - \nabla^2 e = \mu_0 \left(\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{j}\right) = 0, \tag{13}$$

by the conservation of charge, which is no longer automatically satisfied, but must be imposed as an additional law. The scalar field e thus satisfies a sourceless wave equation and propagates with the speed of light in vacuum, not influenced by the presence of charges and currents. This means, in particular, that in boundless space with no incoming "scalar radiation" no e field exists and Maxwell equations have their usual form. This is a particularly attractive feature, as the usual electromagnetism for given distributions of charge and currents in empty space is recovered.

Due to the coupling of the e field with the electromagnetic field, one can expect this scalar field to be generated in the presence of material media that impose conditions on the electromagnetic field, namely good conductors inside which no electric or varying magnetic field exist. Inside such a conductor one has, from Eq. (12d),

$$\boldsymbol{\nabla} e = \mu_0 \mathbf{j}, \tag{14}$$

which acts as a "boundary condition" for the wave equation (13). Note that Eq. (14) makes sense only if the magnetic field in the conductor is zero even inside a current density distribution. This is not a feature of the electromagnetic theory, but has to be assumed in the frame of the extended theory.

In order to apply the extended equations in a medium one should generalize Eqs. (12). For this consider that the propagation of the e field is not affected by the charges or currents present, so one expects no change in its propagation velocity in a material medium due to polarization charges and currents or magnetization currents, in which case e satisfies the same equation as in vacuum. This is ensured if the non-homogeneous Maxwell-Ohmura equations are generalized as

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho + \varepsilon_0 \frac{\partial e}{\partial t}, \tag{15a}$$

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} - \frac{1}{\mu_0} \boldsymbol{\nabla} e, \tag{15b}$$

where again ϱ and $\mathbf{j}$ are the free charge and free current densities.

From Eq.. (15b), taking the time derivative of Eq.. (15a), we see that the contribution to H'_φ, through $\partial \mathbf{D}/\partial t$, from the term $\varepsilon_0 \partial e/\partial t$ is of order λ^2, so it can be neglected in the quasi-stationary approximation considered. In this way, to the effect of the evaluation of H'_φ, $\mathbf{D}$ can be calculated with the original Gauss equation (2). At the same level of approximation the equation (13) for e reduces to $\nabla^2 e = 0$, with the condition (14) inside the conductor. With the usual procedure of integrating the equation $\nabla^2 e = 0$ inside a small "pill box" with one side inside the conductor and the other outside, and employing condition (14) in the interior region, one easily obtains the boundary condition for e just outside the conductor

$$\frac{1}{\mu_0}\boldsymbol{\nabla} e{\cdot}\mathbf{n}_c = -\frac{1}{\delta S}\int_{S_{int}} \mathbf{j}\cdot\mathbf{n} dS = \frac{\partial\sigma}{\partial t},$$

where $\mathbf{n}_c$ is the exterior normal to the conductor surface, S_{int} the surface of the interior region of the pill box, δS its lid surface, and σ the surface charge density. The last equality was obtained using charge conservation. The analogous boundary condition for $\mathbf{D}$ is $\mathbf{D}\cdot\mathbf{n}_c = \sigma$. In this way, in the quasi-stationary approximation, $\partial\mathbf{D}/\partial t$ and $\boldsymbol{\nabla} e/\mu_0$ satisfy the same equation $\boldsymbol{\nabla}\cdot(\partial\mathbf{D}/\partial t) = \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla} e/\mu_0) = 0$ in the space outside the conductors (for this problem $\boldsymbol{\nabla}\times\mathbf{D} = 0$), with also the same boundary conditions. The vector $\partial\mathbf{D}/\partial t - \boldsymbol{\nabla} e/\mu_0$ is thus equal to zero. As a result, from the flux of $\boldsymbol{\nabla}\times\mathbf{H}'$ through a circular surface of radius r at $z = const$ inside the solenoid, using Eq.. (15b) and neglecting the poloidal currents induced in the solenoid (which is correct for large values of ε_{rel} as the charges in the capacitor are distributed mainly in its interior surfaces in this case, with a fraction of order ε_{rel}^{-1} in its outer surfaces and induced on the solenoid surface),

$$r\int_0^{2\pi} H'_\varphi d\varphi = \int\left(\frac{\partial\mathbf{D}}{\partial t} - \frac{1}{\mu_0}\boldsymbol{\nabla} e\right){\cdot}\mathbf{n}dS = 0.$$

As M is proportional to $\int_0^{2\pi} H'_\varphi d\varphi$ integrated in turn over S_o and S_d (see Eq.. (8)) one obtains $M = 0$, eliminating the cause of the anomalous effect.

VI. Conclusions

A relatively simple device, a passive circuit, was hypothesized to present a strange behavior according to the known laws of electromagnetism. The analysis of the processes involved was made at the level of the quasi-stationary approximation which is valid for a realistic range of parameters of the device. The hypothetical increase of energy in this passive circuit does not contradict in principle the conservation of energy, but implies a net Poynting flux entering the circuit. How this incoming flow is effected is the real non-physical aspect of the phenomenon, as not even advanced electromagnetic waves (allowed in principle by Maxwell equations) can be likely responsible due to the non-radiative nature of the effect. Although the inclusion of losses prevents the phenomenon in most cases, a careful design and use of proper materials can lead to a realizable device that could allow to falsify the effect experimentally.

If the analysis presented is correct, one is faced with a paradoxical situation. Two possible resolutions were analyzed neither of which is completely satisfactory, although one expects the first one to be closer to the true resolution. A reconsideration of the equations of electromagnetism is implied by the second proposal. Of course, the extension of Maxwell equations cannot be made without much experimentation and research. The point of this work is to present at least a motivation in this respect. In particular, the extended equations analyzed predict new effects, as the cancellation of the magnetic field inside a charging-discharging capacitor (in the limit of zero resistivity and in a quasi-stationary approximation), which could be tested experimentally.

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Figure caption

Figure 1). a) Sketch of the circuits showing the convention of currents and capacitor charges. The capacitor of each circuit lies inside the solenoid of the other, but they are shown outside in this figure for clarity. b) A cross section of one of the solenoids with the capacitor inside, showing also the dielectric inside the latter, coordinates and symbols used.